Compressible Turbulent Boundary Layer on a Yawed Cone

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Nomenclature

F = Van Driest' wall damping factor (variables are evaluated at the wall values)
i = angle of attack
K',M',Me = $\frac{\bar{w}e}{\bar{u}e}, \frac{1}{\bar{u}e}\frac{\partial \bar{w}e}{\partial \theta}$, Mach number
l = mixing length
m = $(x/\delta)(\partial\delta/\partial x)$
Pr = Prandtl number
Re = Reynolds number per unit length, cm^{-1}
T = dimensionless temperature
u,v,w = dimensionless components of velocity along the x,y, and θ directions
x,y,θ = coordinates, $\theta = \phi \sin\theta_c$
α = $k'w/u$, streamline direction
γ = ratio of specific heats
γ_K = $(1+5.5\eta^6)^{-1}$, intermittency factor
δ = boundary-layer thickness
δ_1^* = $\int_0^\delta (1-(\frac{u^2+(K'w)^2}{1+K'^2})^{1/2})dy$
θ_c = cone half angle
η = similarity variable
ϕ = azimuthal angle
μ = viscosity coefficient

Subscripts

e = external flow condition
t = turbulent condition
w = wall condition

Theme

By using the turbulent viscosity concept and the y/δ similarity assumption, the Prandtl boundary-layer equations are reduced to a parabolic system in only two independent variables (normal and azimuthal). Implicit schemes are used to solve this system. The calculation of efficient and accurate solutions is carried out in the nonseparated region of the boundary layer, from the windward generator up to the cross-flow separation line. Calculations are made for a wide range of Reynolds number values. The locally similar solutions are tested in the symmetry plane with exact solutions. The predictions obtained with four different laws are compared with the experimental results of Rainbird.[7]

Contents

For a weak viscous interaction and due to the conicity of the external flow, the solutions of laminar boundary-layer equations are self similar. In the present case the turbulent terms destroy the similarity along each generator. To again obtain a governing system involving only two independent variables, the usual approach is to consider an integral method. However to study the boundary layer in the case of large incidence it seems to be more appropriate to use the differential method and to seek locally similar solutions as already done by Adams.[1]

The turbulent boundary-layer equations on a yawed cone were derived from the Navier-Stokes equations in a previous paper.[2] For the zero approximation one obtains a system concerning the mean values quantities which is identical to the one derived by Adams. The turbulent terms, i.e., the Reynolds shear stresses and the turbulent diffusion of enthalpy are unknown, and constitute supplementary variables. As a first approach it is thought more convenient to use the isotropic turbulent viscosity and conductivity concept. The expressions of the turbulent viscosity are obtained by a generalization in the three-dimensional case of the semi-empirical laws proposed in the literature. The present analysis is made of the Prandtl's mixing length concept; $\bar{\mu}_t$ reads as:

$$\bar{\mu}_{t1} = \bar{\rho} l^2 F^2 [(\frac{\partial \bar{u}}{\partial y})^2 + (\frac{\partial \bar{w}}{\partial y})^2]^{1/2}$$

The mixing length concept extended to all the boundary-layer thickness is taken following the formulation given by Michel,[3] and is similar to the (Spalding's) one used by Adams.

$$\frac{l}{\delta} = 0.085 \tanh(\frac{0.41}{0.085}\frac{y}{\delta})$$

Smith and Cebeci[4] apply the mixing length only close to the wall (inner region) with the usual form, $l = 0.41\ y$. In most parts of the boundary layer, away from the wall (outer region), the expression suggested by Clauser is preferred.

$$\bar{\mu}_{t2} = 0.168\ \bar{\rho}\ \bar{u}_e\ \delta_1^*\ \gamma_K$$

The validity of the similarity assumption was tested in the plane of symmetry. Comparisons between exact solutions and locally similar solutions show an excellent agreement,[5] particularly with regard to the longitudinal effect on the wall gradients. The similar variable $\eta = y/\delta$ was chosen rather than the Lees Dorodnitsyn's one, used by Adams. As shown in a previous study,[5] it gives a better approximation of the boundary-layer thicknesses. The local similarity assumption was not tested out of the symmetry plane, but this assumption was also used for all the nonseparated region. The locally similar system is then

$$\frac{\partial}{\partial \eta}(\frac{v}{T}) = -(1+m)\frac{u}{T} - (\frac{\partial \delta}{\delta \partial \theta}$$

$$-K'(1+\frac{1+M'}{1+K'^2}M_e^2))\frac{K'w}{T} - \frac{\partial}{\partial \theta}(\frac{K'w}{T})$$

$$\frac{x}{Re\delta^2}\frac{\partial}{\partial \eta}[(\mu+\mu_t)\frac{\partial u}{\partial \eta}]$$

$$= \frac{v}{T}\frac{\partial u}{\partial \eta} + \frac{K'w}{T}(\frac{\partial u}{\partial \theta} + K'(u-w))$$

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Index category: Boundary Layers and Convective Heat Transfer – Turbulent.

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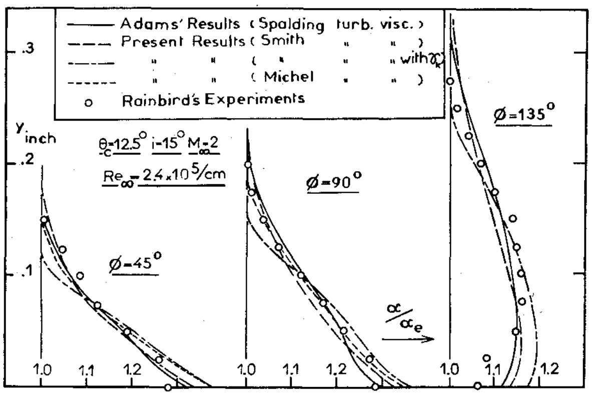

Fig. 1 Profiles of streamline directions within the boundary layer.

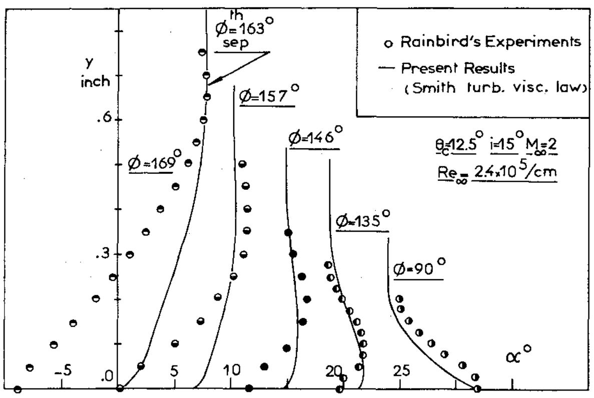

Fig. 2 Flowfield predictions for leeward side of the cone.

$$\frac{x}{Re\delta^2}\frac{\partial}{\partial\eta}\left[(\mu+\mu_t)\frac{\partial w}{\partial\eta}\right]$$

$$=\frac{v}{T}\frac{\partial w}{\partial\eta}+\frac{w}{T}\left(\frac{\partial}{\partial\theta}(K'w)+K'w+u\right)-(1+M')$$

$$\frac{x}{Re\delta^2}\frac{\partial}{\partial\eta}\left[\left(\frac{\mu}{Pr}+\frac{\mu_t}{Pr_t}\right)\frac{\partial T}{\partial\eta}\right]=\frac{v}{T}\frac{\partial T}{\partial\eta}+\frac{K'w}{T}\frac{\partial T}{\partial\theta}$$

$$-\frac{x}{Re\delta^2}(\mu+\mu_t)\left[\left(\frac{\partial u}{\partial\eta}\right)^2+\left(\frac{\partial}{\partial\eta}(K'w)\right)^2\right]\frac{(\gamma-1)}{1+K'^2}M_e^2$$

The boundary conditions are:

$$\eta=0 \quad u=v=w=0 \quad T=\text{constant}$$

$$\eta\to\infty \quad u\to 1 \quad w\to 1 \quad T\to 1$$

The thickness δ is such that $u=0.99$ at $\eta=1$. The laminar viscosity is given by Sutherland's law. The system is resolved by a finite-difference method based on an implicit scheme of the Crank-Nicolson type in the azimuthal direction. Then, after linearization, the finite-difference equations take a tridiagonal form. The nonlinear character of the differential equations, and the adjustment of δ and δ_1^*, lead to an iterative process. Iteration to convergence is used at each azimuth. The solutions calculated at $\theta=0$ provide initial conditions for the solutions out of the symmetry plane. The finite-difference formulas contain a nonuniform grid, based on a geometrical progression of the step size in the normal direction.

In the laminar case, and without viscous interaction, the Prandtl boundary layer exhibits a similar behavior, and the criterion of the cross flow separation is simply given by the condition $\alpha_w=(\bar{w}/\bar{u})$ wall $=0$ (out of the symmetry plane). In fact, as pointed out by Lin and Rubin,[6] the boundary layer may exhibit a nonsimilar behavior in a region inclosing the separation line, even in the laminar case. But, for the conditions used in this study, we will assume that the nonsimilar behavior is not too strong (in the laminar and in the turbulent cases), and we shall use the condition $\alpha_w=0$, as a first approximation, to locate the azimuth of the separation. The step is bissected in the neighborhood of this azimuth and becomes nearly one degree. The separation line is determined for a wide range of x, by using Smith-Cebeci's turbulent viscosity laws without intermittency factor.

The experimental results obtained by Rainbird are used in the present paper to test the method. The external flow parameters are taken from Jones' tables.[8]

The turbulent viscosity laws are tested for $x=100$ cm with $Pr=0.71$ and $Pr_t=0.89$. Comparisons are shown in Fig. 1, concerning the profiles of streamline directions within the boundary layer, for the azimuth values 45°, 90°, and 135°. The profiles predicted by the three viscosity laws, are not substantially different, and agree similarly with Adams' calculations. The agreement with Rainbird's experiments is shown also in Fig. 1, to be better near the windward symmetry plane. For the leeward side of the cone, the flowfield predicted with the law of Smith-Cebeci is given in Fig. 2. Comparisons with experimental data show substantial differences in the neighborhood of the separation. Indeed, in this region, this kind of theoretical approach, which use not only the local similarity, but also the isotropic turbulent viscosity concept, becomes less and less suitable. However these comparisons show that the locus of the separation is reasonably well predicted.

As conclusions to the present study, the following points may be put forward. a) The validity of the local similarity assumption (with the usage of y/δ) has been proven in the windward symmetry plane. This assumption is also used everywhere in the nonseparated boundary-layer flow. b) On the windward side, the boundary-layer flow predictions are, at the measuring accuracy, in good agreement with experiments. Due to the few number of experimental data, it is however, actually impossible to conclude if one among the turbulent models used in this study, is much more efficient. c) When the separated region is approached, the predicted profiles and the experimental data become substantially different. However the azimuth of the separation is, in the present case, well predicted.

A more detailed study would imply a far greater amount of experimental data. In this perspective experimental investigations of the boundary-layer flow are planned at the Institut de Mécanique des Fluides de Marseille.

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